# Energetic coefficient of restitution of binderless elasto-plastic granules

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## Problem

- Distributed micro-mechanical properties throughout the structural volume of a single granule
- Inhomogeneous structure with randomly packed highly poly-disperse primary particles
- Almost always anisotropic and history-dependent behavior

## Goal

- To reduce undesired inelastic deformation of single granules in bulk assemblies by predicting macro-behavior from micro-mechanical properties

## Methodical Approach

- Evaluation of the energetic coefficient of restitution which characterizes the strain energy dissipation i.e. inelastic deformation work of particle contact
- Comprehensive analysis of the trending energetic coefficient of restitution due to dominant influences of:

material properties:
- content
- size
- shape
- density

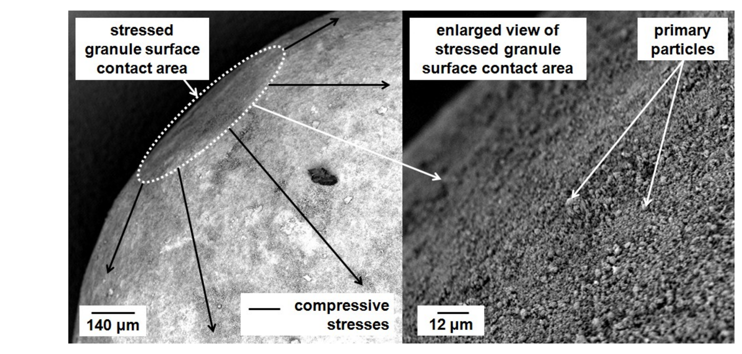
and

process conditions:
- humidity
- stressing intensity
- stressing frequency

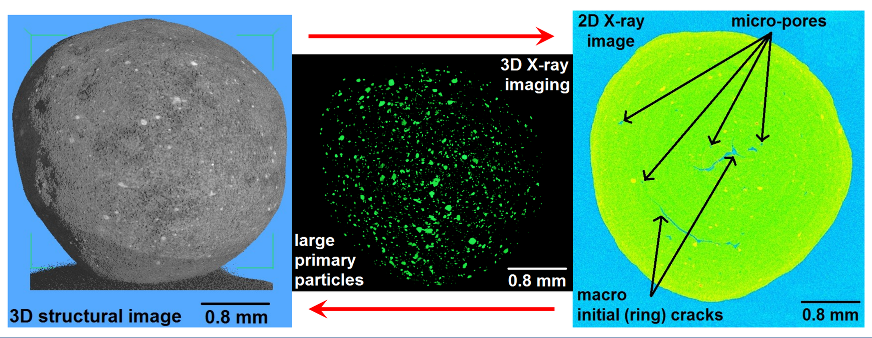
## Model Test Material

- Almost spherical (99%), porous (50–53%), elasto-plastic and binderless millimeter-sized zeolite 4A granules with micron-sized primary particles

SEM:




µ-CT:




## Contact Models

**"Dissipative elasto-plastic"**

Linear elasto-plasticity according to Walton-Braun [1]

$$F_{el-pl,load} = k_{load}\cdot s \qquad (1)$$

$$F_{el-pl,unload} = k_{unload}\cdot(s - s_{max}) \qquad (2)$$

The energetic coefficient of restitution follows [2]

$$e_{E_{W-B}} = \sqrt{\frac{k_{unload}\cdot(s_{max}-s_{unload})^2}{k_{load}\cdot s_{max}^2}} = \sqrt{\frac{k_{load}}{k_{unload}}} \qquad (3)$$

**"Stiff particles with soft healing contacts"**

Non-linear elasticity according to Hertz [3]

$$F_{el,load} = \frac{4}{3}\cdot\frac{E_{gran}}{(1-\nu_{gran}^2)}\cdot\sqrt{R_{gran}\cdot\left(\frac{s}{2}\right)^3} \qquad (4)$$

Non-linear elasto-plasticity according to Tomas [4]

$$F_{el-pl,load} = \frac{1}{2}\cdot\pi\cdot\lambda_{fit}\cdot p_{yield}\cdot R_{gran}\cdot\left(1-\frac{1}{3}\sqrt[3]{\frac{s_{yield}}{s}}\right)\cdot s \qquad (5)$$

Non-linear elasticity according to Hertz [3]

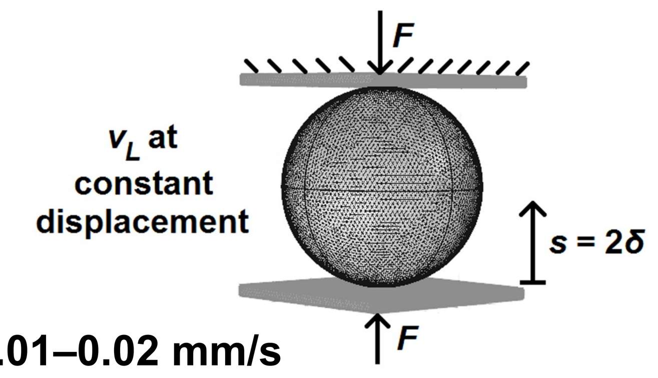
$$F_{el,unload} = \frac{4}{3}\cdot\vartheta_{fit}\cdot\frac{E_{gran}}{(1-\nu_{gran}^2)}\cdot\sqrt{R_{gran}\cdot\left(\frac{s-s_{unload}}{2}\right)^3} \qquad (6)$$

The energetic coefficient of restitution follows [5]

$$e_{E_{H-T}} = 2\cdot\sqrt{\frac{F_{el,unload}}{F_{yield}}}\cdot\sqrt{\frac{s_{max}-s_{unload}}{4s_{yield}+15\kappa_A\frac{s_{max}}{s_{yield}}(s_{max}-s_{yield})}} \qquad (7)$$

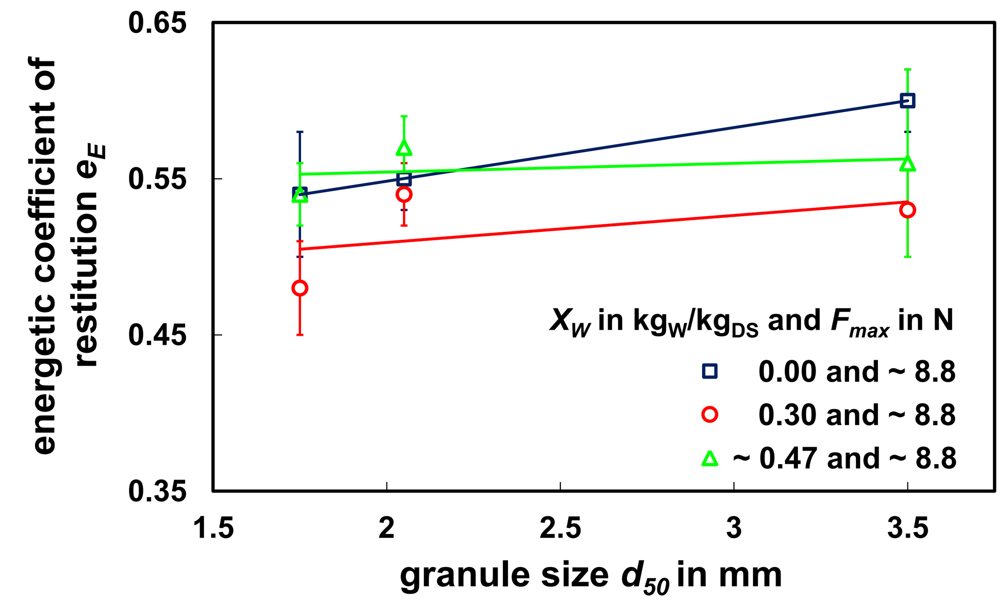
## Test Method

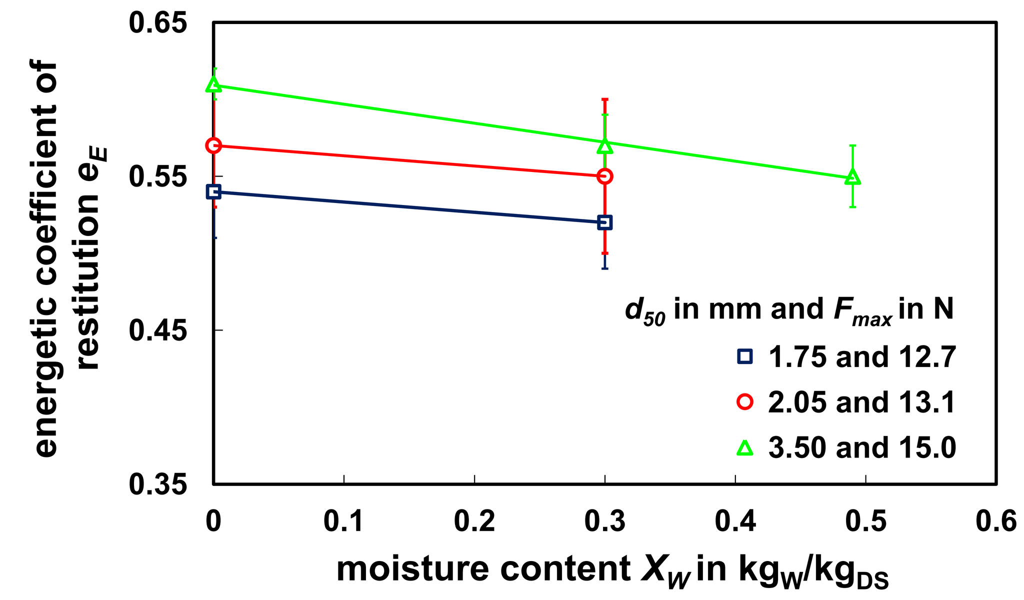
**Quasi-static uniaxial compression**

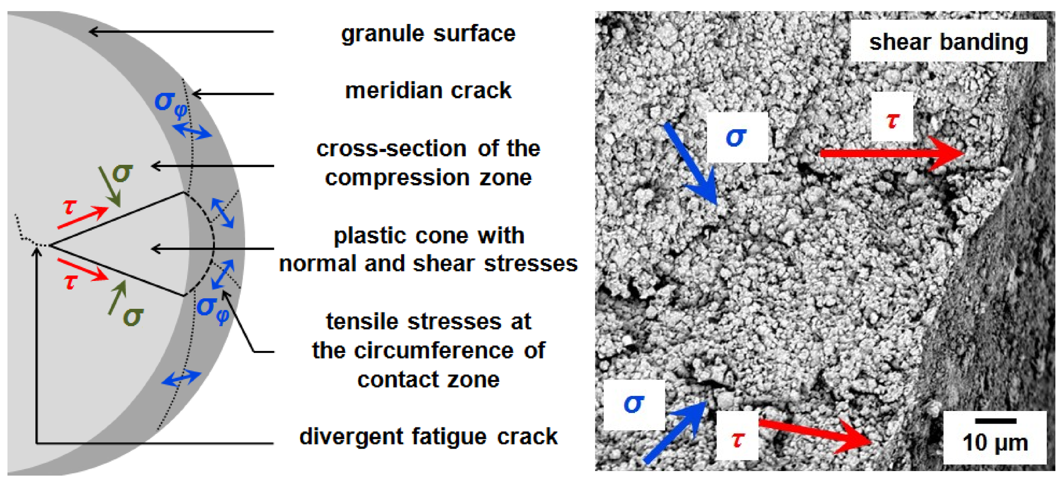


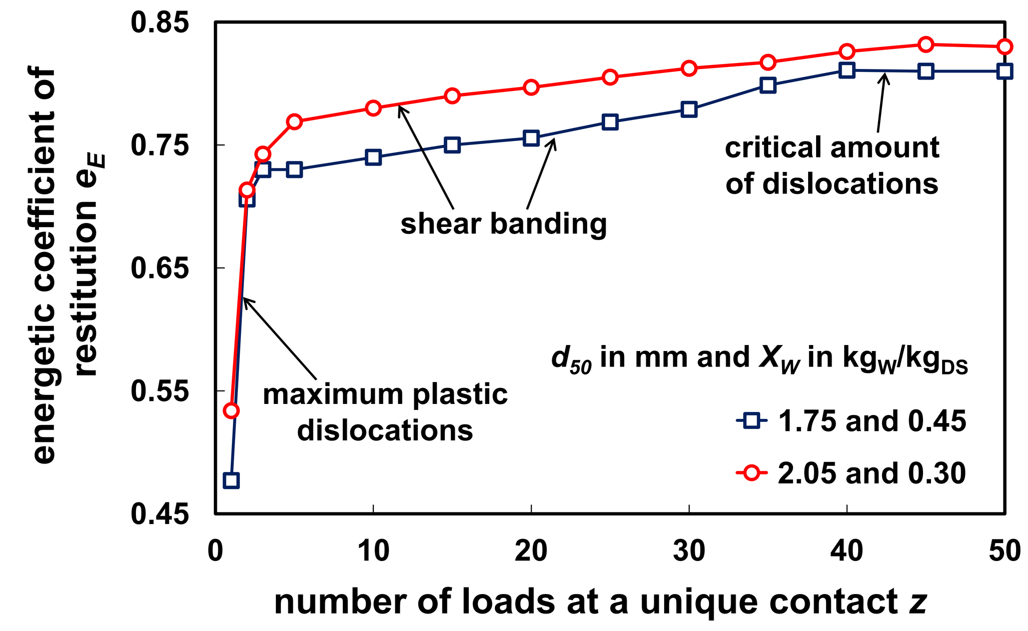

$v_L$ = 0.01–0.02 mm/s

## Results

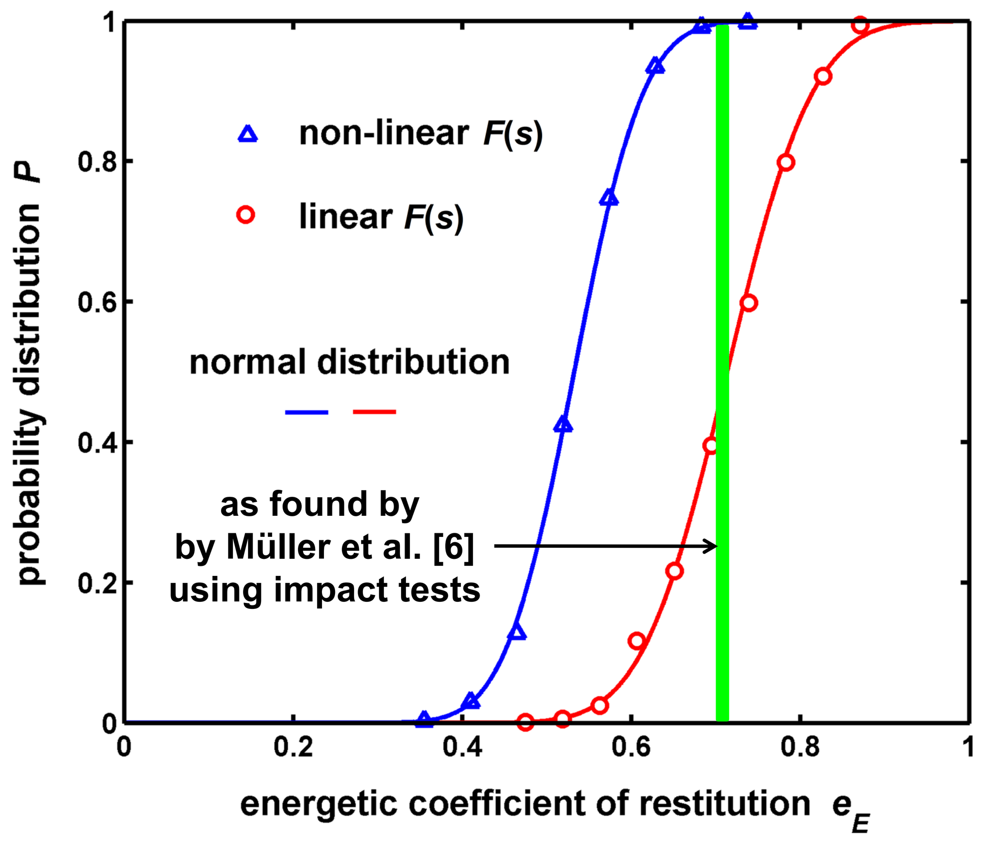


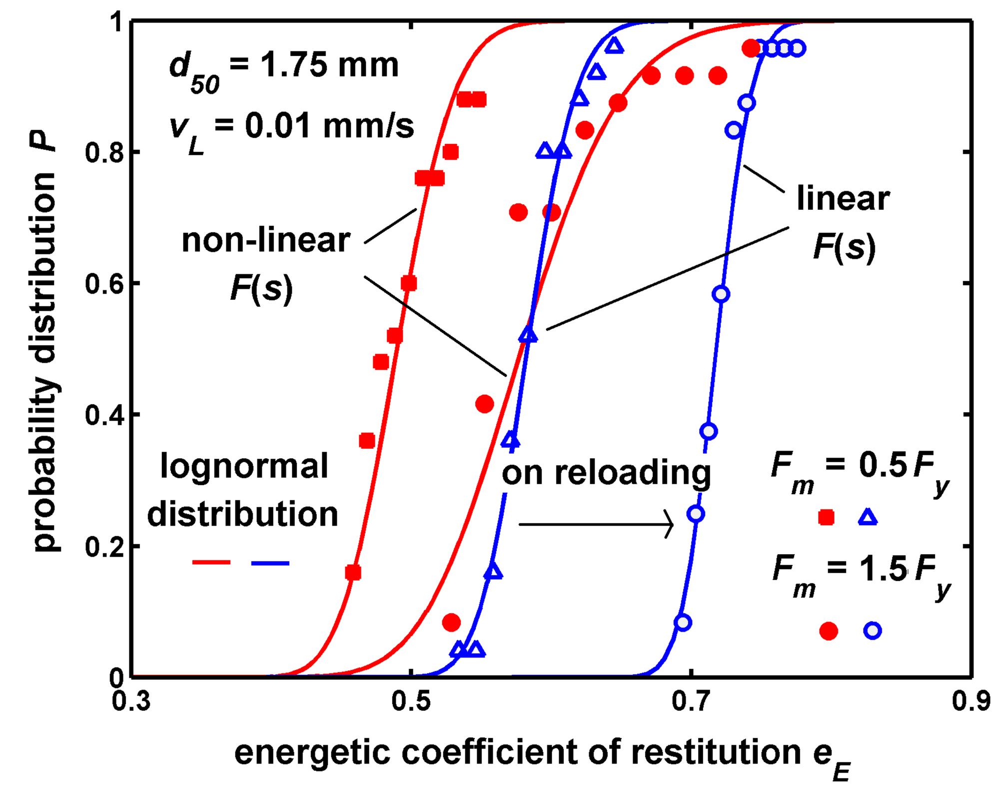




## Results




## Conclusions

- The energetic coefficient of restitution is:
  $e_E$
  - independent of the granule size
  - ∝ 1/moisture content
  - ∝ no. of preloads until crack initiation
- The energetic coefficient of restitution found by compression tests are always lower than the kinematic coefficient of restitution found by impact tests due to the high difference in the "strain rate"

## References

[1] O.R. Walton and R.L. Braun, J. Rheol. 30, 949–980 (1986)
[2] J. Tomas, Vorlesungsmanuskript 1–108 (2014)
[3] H. Hertz, J. Reine Angew. Math. 92, 156–171 (1882)
[4] J. Tomas, Chem. Eng. Sci. 62, 1997–2010 (2007)
[5] A. Russell, P. Müller and J. Tomas, Chem. Eng. Sci. 114, 70–84 (2014)
[6] P. Müller, S. Antonyuk, M. Stasiak, J. Tomas and S. Heinrich, Granul. Matter 13, 455–463 (2011)

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